Linear fit via least-squares (summary)

To fit a straight line ($y = mx + b$) to N data points ((x_1, y_1), (x_2, y_2), $\ldots$, (x_N, y_N)):

$$
\begin{aligned}
m &= \frac{N\left(\sum_i x_i y_i\right) - \left(\sum_i x_i\right)\left(\sum_i y_i\right)}{\Delta} \\
b &= \frac{\left(\sum_i y_i\right)\left(\sum_i x_i^2\right) - \left(\sum_i x_i y_i\right)\left(\sum_i x_i\right)}{\Delta}
\end{aligned}
$$

where

$$\Delta = N\left(\sum_i x_i^2\right) - \left(\sum_i x_i\right)^2$$

To calculate uncertainties in the fit,

$$
\begin{aligned}
\delta m &= \sqrt{\frac{\sigma^2 N}{\Delta}} \\
\delta b &= \sqrt{\frac{\sigma^2\left(\sum_i x_i^2\right)}{\Delta}}
\end{aligned}
$$

where

$$\sigma^2 = \frac{1}{N-2}\left(\sum_i \left(m x_i + b - y_i\right)^2\right)$$

σ^2 can also be calculated via

$$\sigma^2 = \frac{1}{N-2}\left(\left(\sum_i y_i^2\right) - m\left(\sum_i x_i y_i\right) - b\left(\sum_i y_i\right)\right)$$

as long as exact values are used for all quantities — the round-off errors in this formula are *huge*.